

Effective: Fall 2024

COURSE INFORMATION

Course Title: Linear Algebra

Course Number: MATH 232

Credits: 3

Total Weeks: 14 (Fall, Spring)
12 (Summer)

Total Hours: 39

Course Level: ☐ First Year ☒ Second Year
☐ New ☐ Revised Course
☐ Replacement Course

Department: Mathematics

Department Head: G. Belchev

Former Course Code(s) and Number(s) (if applicable): N/A

Pre-requisites (If there are no prerequisites, type NONE): MATH 101 or MATH 111 or equivalent

Co-requisite Statement (List if applicable or type NONE): NONE

Precluded Courses: N/A

COURSE DESCRIPTION

This is a first course in linear algebra. Topics include matrix arithmetic and linear equations and determinants; real vector spaces and linear transformations; inner products and orthogonality; Eigenvalues and Eigenvectors.

LEARNING OUTCOMES

Upon successful completion of the course, students will be able to:

1. Linear systems
 - a. Solve linear systems using row reduction.
 - b. Find the rank of a matrix.
 - c. Answer questions regarding the existence and uniqueness of solutions of linear systems.
 - d. Understand how systems are used to solve problems in science, business and engineering.
 - e. Find the inverse of a matrix using row-reduction.
 - f. Express a system of equations as a vector equation and as a matrix equation and vice versa.
 - g. Solve a system with n equations and n unknowns using
 - i. Cramer's rule
 - ii. The inverse of the coefficient matrix
2. Matrices and matrix operations
 - a. Understand the terms square matrix, symmetric matrix, zero matrix, diagonal matrix, triangular matrix and identity matrix.
 - b. Perform the operations of addition, subtraction, scalar multiplication, multiplication, transpose and inverse of a matrix, and apply the properties of these operations to solve matrix equations.
3. The subspaces of $\mathbb{R}^2$, $\mathbb{R}^3$ and $\mathbb{R}^n$
 - a. Geometric method of vector addition, subtraction, scalar multiplication.
 - b. Understand linear combinations and span of a set of vectors.
 - c. Describe the subspaces of $\mathbb{R}^2$ and $\mathbb{R}^3$.
 - d. Find the vector equation and parametric equations of a line and a plane in $\mathbb{R}^3$.
 - e. Solve problems involving linear combinations, linear dependence, linear independence, the span of a set of vectors, bases and dimension in $\mathbb{R}^n$.

- f. Find a basis and the dimension of the column space and the null space of a matrix.
 - g. Understand the connection between bases and coordinate systems and find the coordinates of a vector relative to a given basis.
4. Inner product, length, distance, angle and orthogonality
 - a. Apply the basic properties of the dot product and use the dot product to solve problems and define the norm of a vector, the angle between two vectors, the distance between two vectors and orthogonality in $\mathbb{R}^n$
 - b. Find a linear equation for a plane in $\mathbb{R}^3$ using a point on the plane and normal vector to the plane.
 - c. Calculate the orthogonal projection of one vector onto another in $\mathbb{R}^n$.
 - d. Use orthogonal projection to find distance of a point from a line and from a plane in $\mathbb{R}^3$.
 - e. Explain the terms standard basis, orthogonal basis and orthonormal basis and be able to convert a basis into an orthonormal basis using the Gram-Schmidt Process (max of three vectors) in $\mathbb{R}^n$.
 - f. Find the orthogonal projection of a vector $\mathbf{y}$ onto a given subspace S of $\mathbb{R}^n$ and find the vector in S that is closest to $\mathbf{y}$.
 - g. Determine the set of least-squares solutions of a given inconsistent linear system.
5. Linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^m$
 - a. Determine the matrices that describe rotation, shear, dilation or contraction and reflection in $\mathbb{R}^2$.
 - b. Matrix transformations, domain, codomain, standard matrix, kernel, range, one-to-one, onto, linearity. Explain these terms in terms of rotation, reflection, etc.
 - c. Determine whether a given transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is linear.
 - d. Determine the standard matrix for a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.
 - e. Form composite of linear transformations.
 - f. Determine the kernel, range, rank and nullity of a linear transformation.
 - g. Determine if a linear transformation is one-to-one.
 - h. Determine if a linear transformation is onto.
 - i. Determine if a linear transformation is invertible, and if it is, find its inverse.
6. Determinants
 - a. Calculate determinants using row operations, column operations, and expansion down any column and across any row.
 - b. Solve a system using Cramer's Rule.
 - c. Find the inverse of a matrix using the adjoint of the matrix.
 - d. Find the volume of a parallelepiped.
 - e. Prove and apply the basic properties of the determinant of a matrix.
 - f. Prove and apply the basic properties of the cross product and use the cross product to calculate the area of a triangle and the volume of a parallelepiped.
7. Eigenvalues and eigenvectors
 - a. Find the characteristic polynomial, eigenvalues and eigenspaces of a square matrix and determine whether the matrix is diagonalizable.
 - b. Find the powers of a diagonalizable matrix.
 - c. Solve problems in population dynamics.
 - d. Solve linear systems of differential equations.
8. Proofs:

Be able to put together a mathematical argument in order to prove simple facts about vectors, matrices, determinants, dot products, length, projection, linear independence, subspaces and linear transformations.

Instructional (Contact) Hours:

Type	Duration
Lecture	39
Seminars/Tutorials	
Laboratory	
Field Experience	
Other (<i>specify</i>):	
Total	39

Grading System: Letter Grades ☒ Percentage ☐ Pass/Fail ☐ Satisfactory/Unsatisfactory ☐ Other ☐

Specify passing grade: 50%

Evaluation Activities and Weighting (total must equal 100%)

	Lab Work: %	Participation: 15% <i>Questions asked in the lecture.</i>	Project: %
Quizzes/Test: 20%	Midterm Exams: 30%	Final Exam: 35%	Other: %

TEXT(S) AND RESOURCE MATERIALS

Provide a full reference for each text and/or resource material and include whether required/not required.

Linear Algebra and its Applications, Latest edition, David C. Lay, Pearson Addison Wesley

COURSE TOPICS

List topics and sequence covered.

Week

Topic

Week 1

Matrices; Matrix Addition; Scalar Multiplications; Transpose, Linear Combinations; Matrix Equations; Applications.
Row-Column Product and General Matrix Product; Matrix Vector Product and its Relation to Linear Combinations and Linear Systems; Properties of Matrix Multiplication.

Week 2

Matrix Multiplication Continued; Vectors in R^2 and R^3 ; Geometric Method of Vector Addition; Subtraction; Scalar Multiplication; Linear Combinations; Span. Subspaces of R^2 and R^3
Vector Equation and Parametric Equations of Lines and Planes.

Week 3

Inner Product, Length, Distance, Angle and Orthogonality;
Scalar Equation of a Plane; Projection; Distance of a Point from a Line/Plane; Orthogonal and Orthonormal Sets of Vectors

Week 4

Matrix Transformations: Determine the Matrices that Describe

Rotation, Shear, Dilation or Contraction and Reflection in R^2
 Explain the Terms Domain, Codomain, Standard Matrix, Kernel,
 Range, One-to-One, Onto, Linearity in Terms of these transformations
 Standard Matrix for a Linear Transformation from R^n to R^m .
 Composite of Linear Transformations

Week 5	Transformations Continued
Week 6	Solving Linear Systems by Row-Reduction, Existence and Uniqueness of Solutions; Rank of a Matrix.
Week 7	Applications of Systems: Polynomial Interpolation; Balancing Chemical Equations; Leontieff's Exchange Model; Network Flow Midterm Exam
Week 8	Applications of Systems: Find the Inverse of a Matrix; Solve Problems involving Linear Combinations; Subspaces of R^n ; Linear Dependence / Independence; Kernel and Range of Linear Transformations; Conditions for being 1-1; onto; Invertible; Inverse of a Linear Transformation.
Week 9	Applications Continued; Basis and Dimension: Row Space; Column Space and Null Space of a Matrix; Subspaces of R^n , Coordinates of a Vector Relative to a Basis.
Week 10	Calculate Determinants using Cofactor Expansion; Row Operations and Column Operations; Properties of Determinants
Week 11	Determinants Continued; Applications of Determinants: Cramer's Rule, Adjoint Formula for Matrix Inverse, Area and Volume.
Week 12	Eigenvalues and Eigenspaces of a Square Matrix; Diagonalization of a Square Matrix, Applications of Diagonalization.
Week 13	Gram-Schmidt Process for Finding an Orthonormal Basis for a Subspace Coordinates Relative to an Orthogonal Basis; Determine the Set of Least-Squares Solutions of a Given Inconsistent Linear System
Week 14	FINAL EXAM

NOTES

1. Students are required to follow all College policies. Policies are available on the website at: [Coquitlam College Policies](#)
2. To find out how this course transfers, visit the BC Transfer Guide at: bctransferguide.ca

Last Revised: September 2024

Last Reviewed: September 2024