COURSE OUTLINE

Effective: Fall 2024

COURSE INFORMATION

Course Title: Linear Algebra

Course Number: MATH 232

Credits: 3

Total Weeks: 14 (Fall, Spring)
12 (Summer)

Total Hours: 39

Course Level: ☐ First Year ☒ Second Year
☐ New ☐ Revised Course
☐ Replacement Course

Department: Mathematics

Department Head: G. Belchev

Former Course Code(s) and Number(s) (if applicable): N/A

Pre-requisites (If there are no prerequisites, type NONE): MATH 101 or MATH 111

Co-requisite Statement (List if applicable or type NONE): NONE

Precluded Courses: N/A

COURSE DESCRIPTION

This is a first course in linear algebra. Topics include matrix arithmetic and linear equations and determinants; real vector spaces and linear transformations; inner products and orthogonality; Eigenvalues and Eigenvectors.

LEARNING OUTCOMES

Upon successful completion of the course, students will be able to:

1. Linear systems
   a. Solve linear systems using row reduction.
   b. Find the rank of a matrix.
   c. Answer questions regarding the existence and uniqueness of solutions of linear systems.
   d. Understand how systems are used to solve problems in science, business and engineering.
   e. Find the inverse of a matrix using row-reduction.
   f. Express a system of equations as a vector equation and as a matrix equation and vice versa.
   g. Solve a system with n equations and n unknowns using
      i. Cramer' rule
      ii. The inverse of the coefficient matrix

2. Matrices and matrix operations
   a. Understand the terms square matrix, symmetric matrix, zero matrix, diagonal matrix, triangular matrix and identity matrix.
   b. Perform the operations of addition, subtraction, scalar multiplication, multiplication, transpose and inverse of a matrix, and apply the properties of these operations to solve matrix equations.

3. The subspaces of R^2, R^3 and R^n
   a. Geometric method of vector addition, subtraction, scalar multiplication.
   b. Understand linear combinations and span of a set of vectors.
   c. Describe the subspaces of R^2 and R^3.
   d. Find the vector equation and parametric equations of a line and a plane in R^3.
   e. Solve problems involving linear combinations, linear dependence, linear independence, the span of a set of vectors, bases and dimension in R^n.

f. Find a basis and the dimension of the column space and the null space of a matrix.
g. Understand the connection between bases and coordinate systems and find the coordinates of a vector relative to a given basis.

4. Inner product, length, distance, angle and orthogonality
   a. Apply the basic properties of the dot product and use the dot product to solve problems and define the norm of a vector, the angle between two vectors, the distance between two vectors and orthogonality in R^n
   b. Find a linear equation for a plane in R^3 using a point on the plane and normal vector to the plane.
   c. Calculate the orthogonal projection of one vector onto another in R^n.
   d. Use orthogonal projection to find distance of a point from a line and from a plane in in R^3.
   e. Explain the terms standard basis, orthogonal basis and orthonormal basis and be able to convert a basis into an orthonormal basis using the Gram-Schmidt Process (max of three vectors) in in R^n.
   f. Find the orthogonal projection of a vector **y** onto a given subspace S of R^n and find the vector in *S* that is closest to **y**.
   g. Determine the set of least-squares solutions of a given inconsistent linear system.

5. Linear transformations from R^n to in R^m
   a. Determine the matrices that describe rotation, shear, dilation or contraction and reflection in R^2.
   b. Matrix transformations, domain, codomain, standard matrix, kernel, range, one-to-one, onto, linearity. Explain these terms in terms of rotation, reflection, etc.
   c. Determine whether a given transformation from R^n to in R^m is linear.
   d. Determine the standard matrix for a linear transformation from R^n to in R^m.
   e. Form composite of linear transformations.
   f. Determine the kernel, range, rank and nullity of a linear transformation.
   g. Determine if a linear transformation is one-to-one.
   h. Determine if a linear transformation is onto.
   i. Determine if a linear transformation is invertible, and if it is, find its inverse.

6. Determinants
   a. Calculate determinants using row operations, column operations, and expansion down any column and across any row.
   b. Solve a system using Cramer's Rule.
   c. Find the inverse of a matrix using the adjoint of the matrix.
   d. Find the volume of a parallelepiped.
   e. Prove and apply the basic properties of the determinant of a matrix.
   f. Prove and apply the basic properties of the cross product and use the cross product to calculate the area of a triangle and the volume of a parallelepiped.

7. Eigenvalues and eigenvectors
   a. Find the characteristic polynomial, eigenvalues and eigenspaces of a square matrix and determine whether the matrix is diagonalizable.
   b. Find the powers of a diagonalizable matrix.
   c. Solve problems in population dynamics.
   d. Solve linear systems of differential equations.

8. Proofs:
   Be able to put together a mathematical argument in order to prove simple facts about vectors, matrices, determinants, dot products, length, projection, linear independence, subspaces and linear transormations.

INSTRUCTION AND GRADING

Instructional (Contact) Hours:

Type	Duration
Lecture	39
Seminars/Tutorials	
Laboratory	
Field Experience	
Other (*specify):*	
Total	39

Grading System: Letter Grades ☒ Percentage ☐ Pass/Fail ☐ Satisfactory/Unsatisfactory ☐ Other ☐

Specify passing grade: 50%

Evaluation Activities and Weighting (total must equal 100%)

	Lab Work: %	Participation: 15% *Questions asked in the lecture.*	Project: %
Quizzes/Test: 20%	Midterm Exams: 30%	Final Exam: 35%	Other: %

TEXT(S) AND RESOURCE MATERIALS

Provide a full reference for each text and/or resource material and include whether required/not required.

Linear Algebra and its Applications, Latest edition, David C. Lay, Pearson Addison Wesley

COURSE TOPICS

List topics and sequence covered.

Week	Topic
Week 1	Matrices; Matrix Addition; Scalar Multiplications; Transpose, Linear Combinations; Matrix Equations; Applications. Row-Column Product and General Matrix Product; Matrix Vector Product and its Relation to Linear Combinations and Linear Systems; Properties of Matrix Multiplication.
Week 2 .	Matrix Multiplication Continued; Vectors in R2 and R3; Geometric Method of Vector Addition; Subtraction; Scalar Multiplication; Linear Combinations; Span. Subspaces of R2 and R3 Vector Equation and Parametric Equations of Lines and Planes.
Week 3	Inner Product, Length, Distance, Angle and Orthogonality; Scalar Equation of a Plane; Projection; Distance of a Point from a Line/Plane; Orthogonal and Orthonormal Sets of Vectors
Week 4	Matrix Transformations: Determine the Matrices that Describe

	Rotation, Shear, Dilation or Contraction and Reflection in R2 Explain the Terms Domain, Codomain, Standard Matrix, Kernel, Range, One-to-One, Onto, Linearity in Terms of these transformations Standard Matrix for a Linear Transformation from Rn to in Rm. Composite of Linear Transformations
Week 5	Transformations Continued
Week 6	Solving Linear Systems by Row-Reduction, Existence and Uniqueness of Solutions; Rank of a Matrix.
Week 7	Applications of Systems: Polynomial Interpolation; Balancing Chemical Equations; Leontieff's Exchange Model; Network Flow **Midterm Exam**
Week 8	Applications of Systems: Find the Inverse of a Matrix; Solve Problems involving Linear Combinations; Subspaces of Rn; Linear Dependence / Independence; Kernel and Range of Linear Transformations; Conditions for being 1-1; onto; Invertible; Inverse of a Linear Transformation.
Week 9	Applications Continued; Basis and Dimension: Row Space; Column Space and Null Space of a Matrix; Subspaces of Rn, Coordinates of a Vector Relative to a Basis.
Week 10	Calculate Determinants using Cofactor Expansion; Row Operations and Column Operations; Properties of Determinants
Week 11	Determinants Continued; Applications of Determinants: Cramer's Rule, Adjoint Formula for Matrix Inverse, Area and Volume.
Week 12	Eigenvalues and Eigenspaces of a Square Matrix; Diagonalization of a Square Matrix, Applications of Diagonalization.
Week 13	Gram-Schmidt Process for Finding an Orthonormal Basis for a Subspace Coordinates Relative to an Orthogonal Basis; Determine the Set of Least-Squares Solutions of a Given Inconsistent Linear System
Week 14	**FINAL EXAM**

NOTES

1. Students are required to follow all College policies. Policies are available on the website at: Coquitlam College Policies
2. To find out how this course transfers, visit the BC Transfer Guide at: bctransferguide.ca

Last Revised: September 2024
Last Reviewed: September 2024